REACH Journal of Special Needs Education in Ireland, Vol. 22.1 (2008), 13–23

Non Routine Mathematical Problem Solving with Lower Achieving Children

This article discusses aspects of a research study which was carried out with twenty-four fourth class pupils in a school designated as disadvantaged in Dublin. The study focused on the strategies children used to solve non-routine mathematical problems, without having prior formal instruction on problem-solving strategies. Within the class were a high number of children performing below the twentieth percentile in mathematics. However, the study found that these lower-achieving children performed beyond expectations on non-routine problems and at times even outperformed their higher-achieving peers.

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BACKGROUND

Problem-solving is central to the aims of the 1999 revised Primary School Curriculum for Mathematics, which advocates that mathematics should "develop problem-solving abilities and a facility for the application of mathematics to everyday life" enabling the child to "plan and implement solutions to problems, in a variety of contexts" (Ireland, 1999, p.12). However, there is increasing concern that a significant number of children may not be equipped with the skills necessary to tackle realistic mathematical problems. The National Assessments of Mathematics Achievement (NAMA) held in 1999 and 2004 found that pupils performed weakest on items assessing higher-order thinking skills, including problem-solving (Surgenor, Shiel, Close and Millar, 2006) and that children in designated disadvantaged schools achieved a significantly lower mean score than pupils in non-disadvantaged schools (pp. 25-29). These findings are supported by the study of Literacy and Numeracy in Disadvantaged Schools (Department of Education and Science (DES), 2005) which indicated that almost two-thirds of pupils in the most disadvantaged schools achieved at or below the twentieth percentile on standardised tests, compared to the one-fifth national average.

The primary purpose of this study was to investigate children's problem-solving strategies in mathematics. Some authors (Wheatley, 1983; Duncan, 1992; Jones, 2003; Foster and Ankers, 2004) recommend children be directly taught problem-solving strategies or follow problem-solving models. Schoenfeld (1992), English (1993), Lester (1994), Trafton and Midgett (2001), Roh (2003) and Windsor (2003) argue against the formal teaching of strategies, agreeing that problem-solving skills, such as strategy development, are learned naturally through solving problems – a position explored as part of this research. The principles of constructivist learning theories were evident in the teaching approaches adopted during the intervention with methodologies based on the belief that children will use and build on existing knowledge and skills in order to solve a problem.

The research was influenced by the author's personal concerns about pupil's problem-solving abilities; in particular their dependence on teachers during problem-solving and their general lack of interest in routine textbook problems. The intervention provided experiences in alternative types of mathematical problem-solving, which were more realistic, motivating and relevant to their lives.

What are non-routine problems?

> *There are five people in a room. Each person shakes hands with every other person once only. How many handshakes are exchanged in the room?*

Non-routine problems, such as the classic *Handshake Problem* above, differ from traditional textbook problems in that the method of solution is not immediately obvious. They can often be solved in different ways and can sometimes have multiple correct solutions. Schoenfeld (1992) describes them as "problems for which there is no standard algorithm for extracting or representing the given information" (p. 357). The underlying principle of such problems is that they involve a process which could help lead the child towards developing a more advanced level of mathematical thinking.

In order to motivate the young problem-solver, many non-routine problems are set in realistic or imaginable contexts – in this study the theme of the popular 2006 Disney production 'High School Musical' was suggested by the class teacher, as the children showed great interest in this film. The thematic approach as influenced by the principles of Realistic Mathematics Education (DeCorte, 1995) facilitates children's active involvement in solving mathematical problems in a context which is either real to them or can be easily imagined. Non-routine problems were sourced and adapted to suit the musical theme (Foster and Ankers, 2004).

THE PARTICIPANTS

A class of twenty-four female pupils participated in the research project over a two month period. At the time, standardised test scores identified seventeen pupils below the twentieth percentile, with four of the remaining seven children scoring between the twentieth and thirtieth percentiles. Twelve pupils attended supplementary teaching for mathematics. Two children had learning difficulties associated with dyslexia and one child was diagnosed with attention deficit hyperactive disorder (ADHD).

A focus group of four children was identified two of whom were regarded as higher-mathematical achievers and two as lower-achievers relative to their class. In fact the two higher-achievers (Natasha and Joanne)[1] only scored at the 32nd and 25th percentiles respectively on their most recent standardised test, with the two lower-achievers (Katie and Tara) scoring at the 14th and 5th percentiles respectively. At the time of the study, these children were not receiving supplementary teaching for mathematics.

[1] Student names used in this article have been changed.

PROCEDURE

During the intervention period, non-routine problems were used weekly with the fourth class pupils. The children were observed as they worked in pairs on the non-routine problems – the focus group working separately in the maths resource room. The author's concern that language and reading difficulties might prevent children from progressing and create feelings of negativity towards the problems was acknowledged in children having the opportunity to ask about new or difficult words.

During each session, the author and the class teacher circulated the room offering support and assistance. Children had access to a variety of resources and manipulatives with a focus on them working independently, developing strategies and justifying their reasoning. The class teacher and author acted as facilitators, encouraging discussion and equal participation without suggesting strategies.

A crucial element of each lesson was its closing discussion where the children were invited to present their strategies and solutions to the class. They demonstrated their chosen strategy, explained why they devised such a strategy and discussed how it worked for them. The class were asked to comment on similarities and differences in the various problem-solving approaches presented. Children frequently commented on how some strategies and methods of record-keeping were more efficient and less time-consuming. Learning to appreciate the approaches and ideas of peers is considered an important part of becoming a successful problem-solver. In later lessons, the children often recalled and applied the ideas and strategies that had arisen during these peer discussions. Following the closing discussion, a home problem was distributed. To build confidence the home problem generally required the application of the strategy used earlier that day.

FINDINGS

This section outlines how the class performed on two particular problems with an additional focus on the strategies of lower-achievers Katie and Tara. *Dance Routines* was presented early in the intervention, while *Six Nations*, which was purposely unrelated to the musical theme, was given eight weeks later to help assess children's retention of strategies.

The Dance Routines Problem

> *There are six girls taking part in one of the dance routines. During the dance each girl must pair-dance with every other girl once only. How many pair-dances will take place altogether? (15)*

Certain characteristics dominated the children's early problem-solving behaviour. They lacked planning and self-evaluation as indicated by their tendency to immediately begin calling out answers. They experienced a low rate of success. About half the class used manipulatives, often colour coded, while the others took a written approach, using names, initials, post-its or colours to identify the dancers. The most common difficulty that occurred was devising a logical method of recording progress. A lack of planning was evident in the random listing approach adopted by most pairs. Children tended not to independently check for duplicates or for missing combinations preventing many from solving the problem in the given time.

One common response to the *Dance Routines* problem was thirty. Children reasoned that if each girl had to dance with five others, and if there were six girls altogether, then six times five is thirty. They were encouraged to re-examine who danced with whom and if the same dance occurred twice, but the lack of an efficient recording system prevented most from revisiting their techniques and meant they had to start again.

At the session's close, the author led a modelling of how the problem might be solved using six pupils and inviting suggestions from the class. Each in turn "danced" briefly with another, while the class kept count. The author frequently stopped and tested the children's understanding of the problem and to recall who had danced with whom.

Lower-achievers, Katie and Tara had an advantage over their classmates as during an initial interview with the focus group several weeks earlier, they had worked with Natasha and Joanne on the *Handshake Problem* using five coloured markers – one marker tapped against another representing a handshake. Initially the children experienced similar difficulties to those of their classmates – in particular keeping track of progress. The group frequently lost count of how many handshakes had taken place and which combinations had occurred. Persistence and co-operation between the four girls led them to decide to use a process of elimination. They confirmed their answer by actually shaking hands with each other enjoying the real-life interpretation of the maths problem.

When Katie and Tara were presented with the *Dance Routines* problem several weeks later, they commented, "It's like the one where we had to shake hands or something". Without any input from the author, the girls solved the problem within minutes, using six green rods, as evidenced below:

Tara (T): (rereads problem and clarifies what is being asked) Oh each girl must dance once only with each girl
Katie (K): (picks up first rod and taps each other green rod)
Both girls count: 1, 2,3,4,5 (puts away 1st rod)
T: Picks up 2nd rod and taps each remaining rod "6, 7, 8, 9" (puts away 2nd rod)
Picks up 3rd rod, taps each remaining rod "10, 11, 12" (puts away 3rd rod)
Picks up 4th rod, taps each remaining rod "13, 14" (puts away 4th rod)
Picks up 5th rod, taps last rod "15"
T: So 15 (girls repeat process to confirm answer).

The girls solved the problem for seven, eight and nine people with ease using the rods. The subsequent challenge to use the emerging pattern rather than the manipulatives to solve for ten people caused them some difficulty.

Researcher (R): Let's have a look again (pointing to page). For 6 people there were 15 dances, for 7 people there were 21 dances. Let's say I didn't let you work out the 28, how could you have used this information to get 28? If I didn't let you use the blocks? (girls look at numbers still confused)
R: For 6 people, there were 15 dances. Let's say I didn't let you use the blocks for 7 people, how might you have figured out the answer? (pause)

K: Because 15 plus 6 is 21
R: Right
K: And 21 and 7 is 28 (shows realisation)
K: and then 9
T: And then 9 and 28 is 36…no
K: Right (starts over) 6 and 15 is 21, 7 and 21 is 28, 28 and 8 is 36, 9 and 36 is…
T: Oh! (enthusiastic) 9 and 36 is 46
K: No, it's 44 (not sure)…45 (checks with fingers)
R: So for 10 people, how many pair dances would take place?
K: 45
R: For 11 people?
Both: (both giggle- unsure what to do)
R: Could you use your pattern to help you?
K: Right 10 and 45 are 55.
T: Ah! 55 and then 11 and 55…
K: For 12 people it would be (uses fingers) 66
T: 11 and 55 (checks also with fingers)…it's 66, is it?

As mentioned earlier Tara and Katie's experience of solving the *Handshake* problem benefited their solving of *Dance Routines*. They independently realised the connection between the two problems and so successfully applied the same strategy to *Dance Routines*. Their classmates' experience of solving *Dance Routines* was different and their success rate was low.

In the early days of the intervention, the majority of children lacked independence and confidence relying heavily on the class teacher and author for guidance and reassurance. Their approaches evidenced a lack of planning, record-keeping and self-evaluation. However, despite the low success rate in the early sessions, the children seemed motivated by this new and alternative type of problem-solving and by the challenge and higher-order thinking required.

The Six Nations Problem

After eight weeks there was evidence of positive progression in the children's problem-solving approaches with significant improvement in levels of persistence. Children ceased calling out random answers, understood that there was a process involved and learned that reflecting on the problem during this process could help to evaluate their progress. They realised that communicating ideas with peers could help the problem-solving process and children frequently consulted neighbouring pairs and commented on each other's ideas and strategies.

The Six Nations Rugby Championship has started. Six teams are taking part: Ireland, Scotland, Wales, England, France and Italy. Each team must play each other team once only. How many games will be played during the Championship? (15)

Children became aware of similarities between problems. Immediately children commented that *Six Nations* was like the *Dance Routines* problem and interestingly when they were pairing up the different teams they frequently used the words “danced with” instead of “played with”. There was a high success rate on this problem. Some children solved this problem very quickly. Others chose to make lists of each game played understanding what was being asked but needing to record all games and then count them. This evidenced an increased awareness of planning. Of the children that listed the teams, all did so systematically beginning with all the teams Ireland played (five), then England (four) and so on. Such use of lists was typical of the children’s new problem solving behaviour. In general strategies became more varied and refined. Along with lists, children began logically recording their work using pictures, diagrams and tables and fewer children depended on manipulatives.

To solve *Six Nations* Katie and Tara initially chose to count the various combinations on their fingers but became confused and changed their strategy imagining the initials of each country.

K: 10, 11, 12…13, 14…15! (solved using imaginary representation of countries)…I don’t know how I got that (smiling), I just knew that I was taking away all the ones (i.e. eliminating one country each time)
R: And does that seem right to you?
T: I don’t think it was because she was just doing it on the table
R: What were you picturing in your head when you were doing that?
K: Well, I could see I, S, W, E, F and I (initials of countries) and then I started with Ireland against Scotland, and that’s where I started counting so it’s 1, 2, 3, 4, 5 (again indicates on table with matching movement) and then I took Scotland and went to Wales, 1, 2, 3, 4
T: 6, 7, 8, 9
K: yeah 6,7,8,9...and then I took Wales and they’re playing now…1, 2, 3
T: 10, 11, 12 (completely following).

The girls commented that the problem reminded them of the *Dance Routines* problem. When the author asked the girls to recall and discuss how they had solved the *Dance Routines* problem, Katie was able to accurately recall the pattern that had emerged at that time:

K: And then the next one would be 21!...no it wouldn’t be 21…hang on 15 add on 6…mmm…I know it but I can’t get it straight into my head…21…and 21 add on 7 is 28…and 28 add on 8 is…I think it’s 34
T: Hang on (uses fingers) 29, 30, 31, 32, 33, 34, 35, 36
K: 36, and then it just keeps going
R: So you’ve remembered a pattern?
K: Yeah because it’s 15 add on 6 is 21.

DISCUSSION

Data analysis revealed positive changes over the eight weeks particularly in communication, logical thinking and evaluation skills improved. The role of the teacher was vital in helping the children to progress these skills. Without directing the children to use a particular strategy, the class teacher and author used careful questioning and discussion, which was primarily focused on raising the children's awareness of their own thinking. Additionally, the children's attention was discreetly drawn to the importance of planning their approaches, recording their progress and evaluating their answers. This interaction between the teacher and children was important in maximising the full learning potential of each problem and enabling the children to experience success, particularly in the early stages of the intervention.

Throughout the intervention, the children were free to choose any resource which would help them. While in the early sessions, counters, links, cubes and post-its were frequently chosen, towards the end of the intervention manipulatives were rarely used. Diagrams and other written approaches such as lists dominated children's approaches. Children began refining their strategies becoming less concerned with the surface features of problems and more concerned with the structural features. An early problem entitled *Costume Design* involved combining different items of clothing to make costumes which many children chose to record using detailed drawings. However as the intervention progressed they began to represent elements of problems in more efficient ways, using symbols and colours. Lester (1994) reports that having the ability to focus on the structural rather than the surface features of a problem is a distinguishing characteristic between good and poor problem-solvers.

As time passed, the children showed higher levels of independence and relied less on the teacher for affirmation. They progressed to the realisation that there was a process involved in solving non-routine problems which would normally be more time-consuming than the process involved in solving routine textbook problems. With success came greater persistence leading to further success and increased mathematical confidence. It was found that persistence was a key quality of the successful problem-solver's behaviour, as children needed to be willing to recognise an unsuccessful strategy and then adapt it or perhaps abandon it to devise a new approach. However, persistence alone did not always lead to success and again the role of the teacher was vital in maintaining levels of motivation. By the end of the intervention, the majority of the class were independently and successfully solving problems. Some children still required more time, but with so many of the class working independently, the class teacher and author could provide more one-to-one attention to these pupils.

Overall the success rate amongst higher and lower-achievers in the class was very balanced. However, lower-achievers often reached solutions first and frequently contributed more useful suggestions for solving problems, some demonstrating a previously unseen level of logical thinking and also an ability to think "outside-the-box". In fact the lower-achieving children often worked at a more practical level opting for manipulatives, while others attempted solutions mentally. Lower-achievers were also more likely to change an unsuccessful approach for a new one and keep a written record of their progress while others displayed an over-reliance on memory.

At times, the higher-achievers seemed more concerned with right answers and how quickly they solved a problem whereas the reverse never occurred. Lower-achievers frequently showed more persistence and remained enthusiastic and determined. Natasha in particular, frequently showed frustration at the long process that was involved in some of the problems. The author reasoned that perhaps higher-achievers were used to experiencing mathematical success in a shorter period of time, whereas lower-achievers may have been more accustomed to experiencing mathematics as a more difficult and slower learning process.

Overall the study evidenced interesting and surprising findings regarding the ability of lower-achieving children to solve complex non-routine problems. Such children's performance was unreflective of the scores achieved on previous standardised tests. Two-thirds of this class scored below the twentieth percentile, with several scoring at the first and second percentiles. The discrepancy between these children's performance on the non-routine problems and their performance on standardised tests raises issues regarding how children's true mathematical ability can be accurately assessed.

CONCLUSION

This study found that children of all abilities were capable of generating their own strategies and solving non-routine problems without prior formal instruction. The author believes that long-term regular use of realistic non-routine problems could help to boost children's mathematical achievement and foster positive attitudes to mathematics. This is particularly important in disadvantaged areas, where mathematical achievement levels continue to be disappointing. The author and class teacher involved in this study were genuinely surprised by what the participating children achieved over the course of a short-term intervention. Children, who had previously scored poorly on standardised tests, performed beyond expectations and were highly motivated by the problems. The non-routine problems promoted a high level of inclusion. Children of all abilities could fully participate. The fact that normally lower-achieving children experienced a new level of success in mathematics had an almost instant effect on self-confidence and on attitudes to mathematics.

Ultimately a combination of factors facilitated the success of the intervention. The use of the thematic approach instantly grasped the children's attention. The questioning role of the teacher was crucial to helping children to progress, to examine their own thinking and to experience success. The interaction between the children and the discussions held at the close of each lesson contributed significantly to the positive progression over the eight weeks. The link with home was an important element of the intervention. Children frequently asked for extra problems to bring home and some began to write their own problems. Parents too became involved and reported that they enjoyed working on the problems in the evenings with their children, commenting that they could see how this alternative type of problem-solving could help to foster logical-thinking skills.

In conclusion, the author strongly recommends that future school planning should begin to embrace alternative approaches to mathematical problem-solving such as non-routine problem-solving.

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