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Problem Solving in the Classroom: Perspectives from the Literature

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> "Solving problems is not only a goal of learning mathematics but also a major means of doing so" (NCTM, 2000, p.4)

INTRODUCTION

A confluence of factors, both societal and pedagogical, has impacted on the role of problem solving in the contemporary classroom. These factors include the rights of students with Special Educational Needs (SEN) to a broad and balanced curriculum (NCCA, 2002; Ireland, 2004), a vision of active citizenship and lifelong learning for all (EU, 2000), vocational imperatives (Xin et al., 2005) and a new focus on developing learner-centred mathematics curricula (DfES, 1999; NCCA, 1999; NCTM, 2000). Notwithstanding its importance, however, problem solving has proven to be a contested and difficult area in mainstream (Surgenor et al., 2006, O'Shea, 2005; NCCA, 2005) and special education (Westwood, 2003; Lerner, 2006).

PURPOSE AND SCOPE OF THIS PAPER

The aim of this paper is to discuss issues from the literature relevant to problem solving in the special education classroom. It will endeavour to survey research in this area and to distil a range of evidence-based classroom approaches that can be used by class teachers.

The paper begins with a brief survey of the literature. Next, a range of classroom approaches will be presented. Finally, a concluding section will seek to summarise key findings and recommendations.

PROBLEM SOLVING IN CONTEXT

Mathematical problem solving is a complex cognitive activity involving a number of processes and strategies (Montague, 2005). It requires students to retrieve previously learned knowledge and apply it in new situations (Bley and Thornton, 2001), using both cognitive and metacognitive processes (Westwood, 2003). In addition, effective problem solving requires significant affective and conative input relating to the student's beliefs and disposition (Jonassen & Tessmer, 1996).

In solving problems, we must decipher both the surface meaning and the underlying task (Barwell, 2003) and activate many mathematical processes including: decision-making, estimating, predicting, calculating and reasoning (Bley and Thornton, 2001). Implicit in this perspective is the belief that there is no single prescribed way to solve problems and that important concepts are actually learned through the problem-solving process itself (Van de Walle, 2004, cited in Lerner 2006).

The next section will consider both the learning opportunities and the challenges for students engaged in solving problems.

PROBLEM SOLVING IN THE CURRICULUM

Problem solving is now an integral part of many curricula (DfES, 1999; NCCA, 1999; Bottge et al., 2002; O'Shea, 2005; Travers, 2005; NCTM, 2000) and its development is considered to be a core aim of the revised Irish curriculum. Working on problem solving activities provides a context in which concepts and skills can be learned through discussion and co-operative working and "is a major means of developing higher-order thinking skills" (NCCA, 1999 p. 8).

DIFFICULTIES WITH PROBLEM SOLVING

In the context of SEN classrooms, Westwood (2003) notes that difficulties with problem solving comprise reading and comprehension issues, failure to apply visualizing strategies, confusion as to which process to use and inability to generalize. Moreover, difficulties in problem representation and failure to identify relevant information and operation may exacerbate their poor performance (Xin et al, 2005).

Other factors contributing to perceived difficulties with problem solving have been noted. The critical role of teachers' beliefs and attitudes in the area of mathematics has been highlighted by an international report commissioned by the NCCA (Conway and Sloan, 2005). In the Irish context, O'Shea (2005) argues that the limited experience of teachers in using problem solving is an obstacle to its effective application.

Bottge et al (2002) indicate that mathematical problems are usually text based, have one correct answer and rarely seem authentic to students. In such a case, Jones (2003, p.87) contends that word problems become "thinly disguised calculations wrapped in words." Additional problems arise with a reliance on whole-class rather than co-operative learning groupings and time constraints that limit discussion and interpretation. In many cases, problems to be solved are taken in isolation so no generic schema is developed in the pupils' minds. With reference to one popular strategy, the keyword approach, Xin and et al (2005, p.181) posit that its use focuses inappropriately on the "surface level of analysis" at the expense of truly understanding the problem. In terms of student engagement, it is noteworthy that in a recent study of self-reported use by sixth class pupils in some Irish schools, Travers (2005) noted the narrow range of problem-solving strategies used.

This section has looked at some issues that impact on problem-solving activities. The next section will look at approaches from the literature that appear to offer support for students in both mainstream and special education settings.

UNBLOCKING THE PROBLEM-SOLVING IMPASSE

Modes of Presentation

Buschman (2005) points out that problem solving can be presented through multiple modalities and not just in the traditional text-based fashion. Additionally, there is a critical need for context and for activating the direct experience of our students

(Bottge et al., 2002; Jones, 2003; O'Shea, 2005). By anchoring instruction (Bottge, 2005) in this way, we can make use of challenging and motivating learning activities to link students' experience to new problems and curriculum content to authentic tasks that have value in the students' life beyond school.

Technology

It appears that technology may act as a valuable enabler in implementing the strategies outlined above. Using appropriate technology, problems can be presented in multiple formats such as, text, image, audio and video which may be more aligned to a student's learning style. For example, the recently launched software package, 'Problem Solvers' (Sherston, 2005) offers text-to-speech, representational support and a graduated help menu to support students.

There appears to be consensus in the literature for the need to move from surface to deep processing (Buschman, 2005; Bottge et al., 2002) by shifting the focus from computation to reasoning (NCTM, 2000). The use of calculators has been proposed by Haylock (2006) as a practical means of allowing students to concentrate on the process of solving problems rather than on calculation per se. Calculator use is recommended in Irish schools from fourth class (NCCA, 1999). Hall (2004), however, cautions that its use should be used as part of a planned teaching process so that students are enabled to build real conceptual understanding.

Classroom Discourse

The benefit of pupils discussing and reflecting on their work has been highlighted by a number of commentators ((Surgenor et al., 2006). Jitendra and Hoff (1996) stress the importance of developing students' mathematical language to support and develop discussion and reflection. In addition, Pugalee (2004) contends that students who have opportunities to engage in mathematical communication receive a dual advantage of communicating to learn mathematics and learning to communicate mathematically. In general, we can say that discussion helps to clarify thinking and deepen understanding of concepts and allows pupils to see that there are many approaches to solving a given problem (Checkley, 2006).

Problem Representation

The ability to represent problems visually helps pupils to externalise their conceptual understanding. From the perspective of the teacher, this visual representation serves as a window to the pupil's thinking and understanding. Moreover, effective visual representation shows the relationships in the given problem and acts as a scaffold for understanding and explanation. Poor problem solvers tend to make immature representations that are more pictorial than schematic in nature (Montague, 2005). This is due to the fact that pupils who draw pictures to conceptualise problems are inclined to focus on non-essential features. This viewpoint is supported by Hegarty and Kozhevnikov's study (1999), which showed a connection between successful problem solving and schematic rather than pictorial images.

Cognitive and Metacognitive Strategies

In the SEN classroom, direct instruction in cognitive and metacognitive strategy use is recommended (Lerner, 2006). Westwood (2003, p.191) advocates teacher modelling of effective strategy use through 'think alouds' as he is working. The cognitive processes and strategies needed for successful mathematical problem-solving include paraphrasing, hypothesizing, estimating, computing and checking (Montague, 2005).

Schema-based instruction (SBI) is a cognitive approach that allows the learner to visualize a representation to solve a range of different, but structurally similar problems of a particular. Research findings demonstrate that those pupils taught with SBI performed better that those in a control group in acquisition, maintenance and generalization type (Jitendra et al., 2005a).

Assessment

Assessment is increasingly being seen as a way of informing and improving learning. Assessment that merely require students to use well-rehearsed procedures in solving predictable problem types do little to assess the degree of learners' strategic knowledge. Rather, assessment needs to focus on the underlying higher-order thinking that characterizes problem solving. Otherwise, this thinking will remain largely invisible and unacknowledged. Instruments that seek to value and assess student thinking include the use of paper-based and electronic portfolios (O'Callaghan, 2005), structured observations of students, student interviews, rubrics, concept maps and student journals.

This section has investigated possible approaches to improving the problem-solving abilities of our students. The summary of classroom strategies listed below are based on these findings from the literature, on recommendations in Irish curricular documents (NCCA, 1999; 2002) and on social constructivist learning models.

CLASSROOM STRATEGIES

Planning

- Problem types will be introduced beginning with the easiest problem type, and after mastery, move to the next level of difficulty (Montague, 2005)
- A spiral approach will be used as generic problem types are revisited and students' mental schemata are developed.
- The teacher will use a combination of explicit instructions, such as process modelling, and guided discovery approaches to classroom activities.
- The teacher will demonstrate and model what good problem solvers, using 'talk aloud' strategies
- There will be an emphasis on problems that can be adapted for multiple levels. These are referred to as 'low threshold-high ceiling' problems because of their low entry level coupled with challenging opportunities for the more able.
- ICT will be used as a bridge between the enactive and the symbolic stages of learning (Bruner, 1966)
- The development of metacognitive skills will be considered an essential component with an emphasis on the following:
    - Planning the steps necessary to complete the task.
    - Sequencing the steps.
    - Self-monitoring as one progresses through the process (Bender, 2002)

Presentation

- Problems to be solved will be presented in multiple modalities.

- The prior knowledge of students will be considered an essential component of the problem-solving task.
- Students will be encouraged to restate the problem in their own words.
- The students will work collaboratively in pairs and small groups.
- Problems are tackled as they occur, for example in the PE class or during a cookery session, in addition to the maths class.

Student Engagement

- Students will have the choice of using calculators in order to concentrate on reasoning rather than computation.
- Students will be encouraged to represent their conceptual understanding through schematic drawings, concept mapping etc.
- Discussion will be prioritised prior to, during and after each problem solving task in order to support students' reasoning skills and use of mathematical language. These learning dialogues will help to clarify thinking and help students see that there are many different approaches to solving problems.

Reflection and Assessment

- Assessment will be used to inform classroom practice.
- The teacher will formatively assess and evaluate the approaches, strategies and processes that the students use in these tasks.
- Students will self-reflect using an evaluative rubric after each class.

CLASSROOM PRACTICE

For reasons of space, this section presents one abbreviated exemplar lesson plan involving a 'low threshold –high ceiling' problem of the 'change' type (Hayes, 2003). The class comprise adolescent students in a special school setting.

Learning Targets

The students will learn and practise appropriate cognitive steps in problem solving. They will then solve other problems with a similar mathematical structure in other contexts.

The pupils will verbalise and share their problem-solving strategies

Materials

Interactive whiteboard/Computer Software
Counters and manipulatives/Calculators
Cue sheets with step-by-step help for some students
Self-check Rubric

Methodology

- The problem is presented orally and on the whiteboard.
- Elicitation of student experience and strategies for solving this problem.
- The pupils represent the problems schematically.
- Teacher and pupils model appropriate strategies through 'talk alouds'
- Appropriate cues and prompts are offered as students learn and practise.

LESSON SEQUENCE

The pupils will represent the problem, using the change frame[1], and solve it using a mathematical sentence.

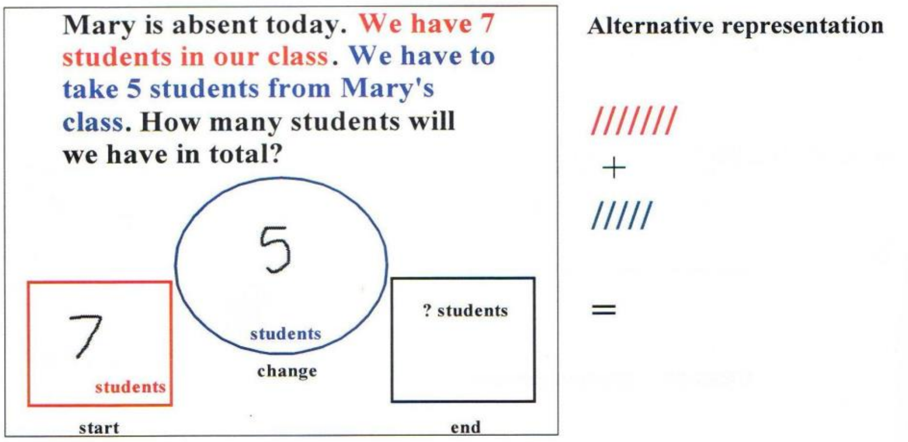

Figure 1

Figure 1

Sample Classroom Discourse
"Is this a difficult problem?"
"Does it involve adding or taking away?"
"How did you work out the answer?"
"How many different ways did we find?"
"Can you think of another problem like this?"
"Can you think of a way of remembering how to solve this type of problem?"

Differentiation

Students who need more scaffolded practice may benefit from using appropriate technology to work on problems with a similar structure. The screenshots below (Figures 2, 3) show the problem presentation (with speech output) and the four-stage graduated help screen[2]

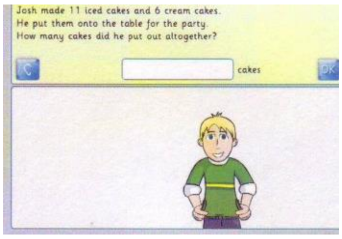

Figure 2

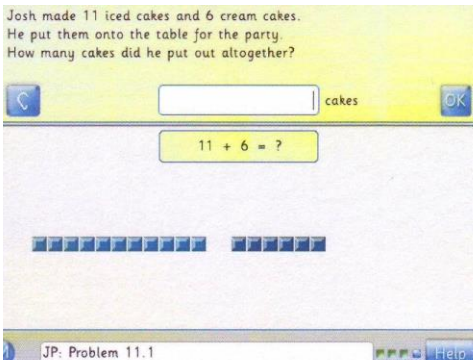

Figure 3

Students who need more challenge can construct and solve other problems with a similar structure using a simple teacher-designed drop-down form (Figure 4).

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[1] Based on diagram by Jitendra et al (2005a)

[2] 'Problem Solvers' – Semerc Software

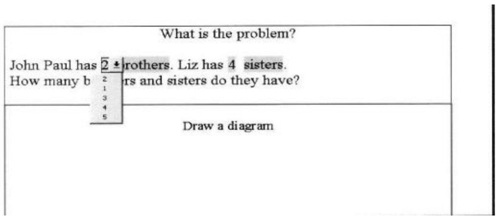

Figure 4

To encourage independence, a guide sheet (Table1) is used by the participants.

Solving Problems		
1	**Read** or **listen** to the problem	☐
2	Tell the problem **in my own words** to my partner	☐
3	Draw a **diagram** to show the problem	☐
4	**Explain** my diagram to my partner	☐
5	Do I need to add or subtract? **Explain** to my partner.	☐
6	**Write a maths sentence**	☐
7	**Guess** the answer	☐
8	**Work out the answer** – use your diagram / 100 square / calculator to help	☐
9	Does my answer make sense? **Explain** to my partner.	☐
10	**Check** my answer with my partner	☐

Table 1

Assessment

The teacher will assess the students' understanding informally by listening to their 'talk aloud' strategies as they work, by analysing the self-check rubrics (Table 2), as well as by examining the software logs for each user.

	4	**3**	**2**	**1**
My Strategy	I retold the problem and drew a diagram. I worked it out and checked to see if it made sense. ☐	I drew a picture and underlined the words. Then I worked it out. ☐	I looked at the numbers and worked on them. ☐	I guessed the answer. ☐
My Work	I thought about the problem and tried to fit it to one that I worked on before. ☐	I thought about the problem and figured out what type it was. ☐	I worked on the numbers. ☐	I did not check anything. ☐
Teamwork	I listened to my partner. I did my own work and	I listened to my partner and did my own work.	I did my own work. I did not listen to or help my partner.	I did not help anyone.

	helped my partner too. □	□	□	□

Table 2

CONCLUSION

> "Problem solving should be the site in which all of the strands of mathematical proficiency converge." (National Research Council, 2001, p. 421)

Problem solving is generally regarded as the most important cognitive activity in our everyday lives. (Jonassen, 2000). Therefore, the need to identify principles of good practice in this area is paramount.

The need for debate and action has been highlighted in the recently-published report by the DES (Surgenor et al., 2006) where the achievement of fourth class students in the skill category of Applying and Problem Solving compared unfavourably to results in other areas. The report highlights recommendations made by DES inspectors for more use of authentic contexts in problem solving and for more systematic and explicit approaches to teaching problem solving.

Jonassen (2000) maintains that our students will suffer if we neglect the real problem-solving challenges that await them outside of the confines of the classroom. Over-reliance on solving the well-structured problems that we find in most textbooks is little preparation for the type of complex, ill-structured and multi-faceted problems they will need to solve in their everyday lives.

The message from the literature appears to be clear and compelling: problem solving, as a classroom activity, needs to break free from the confines of the Mathematics textbook and occupy a more central, cross-curricular and discourse-rich position.

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